TABLE 1.—*Number of States*[1] *in which auroras were reported as observed on any given date in 1919.*

1919.	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec	
1	1	1	1	1	4	1		1	1	11			
2	1		1		10	2		2	2	5	1		
3	2				6					2	1		
4	4	1			2	2		1		2			
5	2		1		4						1		
6		1	2	2	1			1					
7			1					1		2			
8	1		1		1				1	1			
9	1			2					1				
0	1								1	1	1		
11								8			1		
12			1		1	1		1			1		
13	1	3	1							1			
14					1			1				1	
15			1						5	3	1	1	
16		1	2	1	2				1	1	3		
17	1		1		1			1	3	5	2		
18				3	1			2	7	1		1	
19		1	3	3	1		1	1	8		1	1	
10		4	6	2	2			1	5		1	1	
21		4	7	4	1			1	2		1		
22	1	4		4	1	2			2	5			
23		1	1	1				1	7	6	2		
24			1	1	1	1		1	10	1			
25			2	1	1	2		1	5				
26		4	2		1			1	3	2			
27		8	3	1		1		1	1	1			
28	2	7	2			1		1		1			
29									1	3	1	1	
20	1		1						1	1			
31	3				1					2			
Number of days with auroras	14	13	21	13	20	9	1	19	20	21	14	6	171

[1] For the purposes of this table, the District of Columbia is regarded as a State.

2.1 days throughout the year, taking, of course, the country over. In point of seasonal distribution September seems to have been the most favored month, while July was the least.

Table 2 is designed to give a further idea of the seasonal extent of the auroras observed.

Thus no less than 28 such displays were seen in at least four States at one time, while three displays were witnessed in 10 States simultaneously.

Table 2 also shows the number of days on which auroras were seen. In this connection it is interesting to note the distribution (in point of time) of the auroral displays. The first quarter of the year had a total of 48, the second quarter, 42, the third quarter, 40, and the last quarter, 41. March and October had the largest number of displays—21 each. July had the least—only one.

TABLE 2.—*Number of auroras in 1919, arranged by State groups*

Number of States	4	6	8	10
Number of auroras	28	13	6	3

It is not practicable to go into any details of these auroras here, but any one wishing to study accounts of of the more prominent ones may apply to the Weather Bureau library for information.

A SIMPLE EQUATION OF GENERAL APPLICATION FOR THE NORMAL TEMPERATURE IN TERMS OF THE TIME OF DAY AND THE DAY OF THE YEAR.

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(Logan, Utah, July 16, 1920.)

SYNOPSIS.

The following empirical equation

$$T=\frac{Ma}{2}+\text{Va}\cos t+\frac{My}{2}\cos\theta$$

represents the normal temperature as a function of the time for the United States except for the arid West, where we must add the term $\left(\frac{\text{Vv}}{4}\cos t\cos\theta\right)$. The constants are the mean annual temperature, the range of the annual march, and the range of the daily march, and are obviously easily obtained from the Weather Bureau data for the place desired. The mean error for the arid West was 2.75° F. and it is less for the rest of the United States. The equation simply assumes that the annual and daily march of temperatures are simple cosine functions.

DISCUSSION.

It is generally known that the air is alternately warmed during the day and cooled at night and also warmed in summer and cooled in winter. The normal temperature then is a periodic function of the time with a twenty-four hour and an annual period. The writer[2] obtained an empirical equation for these changes with the aid of the Fourier analysis and called attention to the fact that these series, made up of cosine terms, converged so rapidly that the omission of all of the terms except the first two had only a slight effect on the result. Professor Marvin[3] also found that in some regions the annual march of temperature is very approximately represented by a simple cosine curve. Its equation would therefore be

$$Md=Ma+\frac{Va}{2}\cos t \qquad (1)$$

where Ma represents the mean annual temperature;
Md represents the mean daily temperature for the day t;
Va represents the annual range in temperature or the difference in the mean daily temperatures of the hottest and coldest day of the year, and
t represents the time of the year, zero of time being on the hottest day of the year or at the maximum of the curve.

[1] The writer received helpful suggestions from Dr. Willard Gardner, Associate Physicist, Utah Agricultural Experiment Station.

[2] West, Frank L., Edlefsen, N. E., and Ewing, Scott—"Determination of the Normal Temperature by Means of the Equation of the Seasonal Temperature Variation and of a Modified Thermograph Record." (*In* Physical Review, Vol. 14, No. 3 (1919). Also *in* Jour. Agr. Rsch., Vol. 18, No. 10 (Feb., 1920). *Abstract in* MONTHLY WEATHER REVIEW, Dec., 1919, 47:877.

[3] "Marvin, C. F.—Are Irregularities in the Annual March of Temperature Persistent?" (*In* MONTHLY WEATHER REVIEW, Vol. 47, No. 8, p. 544 (1919).

This paper indicates that for a section of the northeastern United States represented by New England and the States of New York, Eastern Ohio, Pennsylvania, Maryland, and part of Virginia, the annual curve of temperature is well represented by a single cosine curve. Elsewhere in the United States two terms are in general required with an amplitude of the second term of 1 to 2 or more degrees. In the arid southwest later studies show three terms are necessary for consistent accuracy.

The literature on the subject is best reviewed in the two following articles: Pernter, J. M.—"Present Status of Our Knowledge of the Causes of the Diurnal Changes in Temperature, Pressure, and Wind." (*In* MONTHLY WEATHER REVIEW, Vol. 42, No. 12, pp. 655-665 (1914).

Talman, C. Fitzhugh—"Literature concerning Supposed Irregularities in the Annual March of Temperature." (*In* MONTHLY WEATHER REVIEW, Vol. 47, No. 8, pp. 555-565 (1919).

The curve representing the twenty-four hour temperature change (thermograph record) modifies its shape gradually each day, flattening out (to an extent depending upon the locality) as winter approaches. During the summer when the heat is being received fast, one would expect the rise in temperature to be greater for the same time-interval than in winter, and thus in arid sections it is found that the daily variation in temperature is nearly twice as much in summer as in winter. However, for regions of small rainfall the ratio of the hourly temperatures to the mean daily temperature (Fahrenheit) is nearly constant whatever day of the year is selected, e. g., the ratio of the maximum to the mean is approximately constant for all days of the year. Irregularities will be mainly caused by storms and enough ratios must be taken in obtaining this constant to eliminate these. The plot showing the percentages of the mean for each hour is approximately a cosine curve and has the following equation:

$$P = 100 + \frac{Vp}{2} \cos \theta \qquad (2)$$

where P represents the per cent of the mean that the hourly temperature has, Vp represents the total variation or range in the per cent for the 24 hours, θ represents the time of day expressed in degrees.

Since the hourly normal would be the per cent of the mean daily multiplied by the mean daily temperature, it would be equal to the product of equations (1) and (2), giving

$$T = P\ Md = \left(100 + \frac{Vp}{2} \cos \theta\right) \left(Ma + \frac{Va}{2} \cos t\right) \qquad (6)$$

This equation will give fairly accurate results for normals of the arid regions, but falls down in humid sections because as winter approaches the daily range does not diminish as fast as the mean daily does, thus making this constant really a function of the time, or, in other words, not a constant at all. It is not quite rigorously true in the arid regions because if it were the range and the temperature would have to vanish together, i. e., when in winter the mean daily temperature reached 0° F. there would have to be no daily range but a constant temperature for the 24 hours. From 20° F. up it works very satisfactorily, however, and in the arid west of this country probably 95 per cent or more of the days of the year have mean daily temperatures higher than this; hence, it gives satisfactory results for this part of the country.

Another somewhat similar equation will now be derived which fits the facts still better and is more general in application.

The normal thermograph record for a particular day is approximately a cosine curve. In fact, it will be shown later that if the time or angle be counted from the maximum and the minimum on the curve, that the following simple equation will fit the facts even better than the corresponding one fits the annual range in temperature:

$$T = Md + \frac{Vd}{2} \cos \theta \qquad (3)$$

where T represents the temperature at the time of day θ, the hours being expressed in degrees,

Md represents the mean daily temperature,

Vd represents the daily variation or range in temperature for that day.

Vd, or the daily variation in temperature, is also a periodic function of the time, the period being one year and the maximum occurring in the summer, and for the reasons indicated earlier in this paper.[4] The difference between the daily range in summer and in winter is from 0 to 5 degrees for points in the United States outside of the arid West and about 10 degrees for the latter section. The following equation represents fairly well these facts:[5]

$$Vd = My + \frac{Vv}{2} \cos t \qquad (4)$$

where Vd represents the daily range for the day t,

My represents the average daily range for the year,

Vv represents the difference between the range in summer and winter.

Substituting equations (1) and (4) in equation (3) the following equation is obtained for the normal temperature T on the day t and for the hour θ:

$$T = Ma + \frac{Va}{2} \cos t + \frac{My}{2} \cos \theta + \frac{Vv}{4} \cos \theta \cos t \qquad (5)$$

This equation is a general one and the constants for a particular city can be obtained in less than five minutes' consultation of the U. S. Weather Record for that place. For all places except very arid sections the last term is zero and the equation is thus still simpler.

This equation rests on the assumption that the annual march of temperature can be represented by a single cosine function, that the daily march can be represented by a single cosine function, and that the change in the daily range with the season can likewise be represented by a cosine function. Although there is a physical reason for each of these periodic changes, yet the above equation is purely empirical and meritorious only to the extent that it fits the facts and is simple of application. These marches of temperature are not exactly cosine functions for the curves are not quite symmetrical, the number of days between the minimum (about Jan. 15) and the maximum (about Aug. 1) for the year counted in the spring being usually somewhat more than when counted in the fall. Likewise, the time-interval between the minimum in the morning (about 6 a. m.) and the maximum (about 3 p. m.) is about nine hours measured in the morning, while it is fifteen hours when measured through the night.[6] This error, however, may be largely eliminated by making the curve pass through both the maximum and the minimum in the process of converting the days and hours into degrees before substituting in the equation, e. g., for Salt Lake City, or most any city of the same latitude, an hour in the morning represents 20°, $\frac{180}{9}$, while in the evening it stands for only 12°, $\frac{180}{15}$; and a day in the spring of the year 0.9 of a degree, and one in the fall 1.1°. Thus, starting time when the temperature is a maximum (3 p. m. for Salt Lake City) the angle corresponding to 9 a. m., which is three hours

[4] "Climatology of the United States," pp. 94–97 (*In* Bulletin Q, U. S. Weather Bureau)

[5] Lack of being an exact cosine function is not at all serious because this term is negligible in most regions. See fourth column of Table No. 1. Where it is appreciable it followed the cosine law very well.

[6] By replacing cos t by $\cos\left(\frac{2\pi t}{365} + 10 \sin \frac{\pi t}{365}\right)$ and cos θ by $\cos\left(\frac{2\pi\theta}{24} - 30 \sin \frac{\pi\theta}{24}\right)$, the equation will fit the facts better, but it becomes now nearly as involved as the Fourier series equation, but still has more readily determined coefficients.

after the minimum, would be $180+3\times 20=240$ rather than $\frac{360}{24}\times 15=225°$, and 5 a. m. would be $180-12=168°$, 2 p. m. $=360-20°$, and 4 p. m. $=12°$. The lack of symmetry of these curves, therefore, can be largely eliminated by counting the time from the nearest maximum or minimum of the curve.[7]

The equation for Salt Lake City becomes

$$T=51.5+22.2 \cos t+10 \cos \theta+2.55 \cos \theta\, t,$$

the addition of the first two terms giving the mean daily temperature for the day *t*, and the sum of the four terms the normal hourly temperature for the hour θ on that day.

Assuming that the normal hourly temperatures for the month represent the normal thermograph record for the 15th of the month, these monthly values were obtained from as many U. S. Weather Bureau stations as possible and checked against the values obtained from the corresponding equation for that city for the 15th of each month. The mean error ± for Salt Lake City was 2.7°, for Portland, Oreg., 2.2°, for San Francisco, 2.4°, for Chicago, 2.5°, for New York City, 2.8° F., for Memphis, Tenn., 2.2° F. Using the first two terms only and thus obtaining the mean daily temperatures the errors were 1.37°, 1.4°, 1.6°, and 1.2° F., 2.0°, and 0.5° F., respectively, in this determination of mean daily temperatures.

Table 1 contains the constants for equation (5) for 24 rather widely distributed representative cities of the United States. For all places except the arid West the last constant (see column 4) is 1 or less, and when multiplied by 2 cosines it becomes negligible; hence, the equation is still further simplified, becoming

$$T=Ma+\frac{Va}{2}\cos t+\frac{My}{2}\cos \theta \qquad (8)$$

The U. S. Weather Bureau has hundreds of cooperative weather stations where thermograph records are not kept. The data taken at these stations are sufficient to determine the constants of equation (5), and thus this equation can be used for determining hourly normal temperatures for these places.

About one-fourth of the earth's surface has a relative *humidity of about 50 per cent; 80 per cent of the* days are free from rain; the precipitation is from 10 to 20 inches; the sky clear most of the time; and hence the departures from the normals are comparatively small as the records show. Equations (5) or (8), therefore, will give *actual* temperatures with fair accuracy for dry areas. Further accuracy may be obtained with the aid of the weather forecast.[8]

In using this equation for the determination of minimum temperatures at smudging time it should be remembered that even though a frost usually occurs on a clear night when the thermograph record is likely to be uniform yet the frosts that do damage to crops are usually abnormal, following cyclones, while the equation gives normal temperatures. These normals from the equation considered with the observed departures from the normals during the preceding 24 hours should give a fairly accurate prediction of the minimum to be expected.[9]

TABLE 1.—*Constants for the equation $T=Ma+\frac{Va}{2}\cos t+\frac{My}{2}\cos \theta+\frac{Vv}{4}\cos \theta \cos t$ for various cities of the United States.*

City.	Mean annual temperature = Ma.	Range of the annual march $\div 2=\frac{Va}{2}$.	Average daily range $\div 2=\frac{Mv}{2}$.	Range of daily range $\div 4=\frac{Vv}{4}$.	Average daily variability of temperature.
East Atlantic States:					
Boston	49	23	8	0.5	5.6
Albany	48	25	8	1.0	....
Buffalo	47	23	6	0.7	....
New York	52	25	7	0.7	....
Washington	55	22	9	6.0	4.8
Atlanta	61	16	8	0.7	4.1
Jacksonville	69	14	8	0.5	3.5
Central States:					
Little Rock	62	20	9	1.0	....
New Orleans	69	18	7	0.5	3.3
Galveston	70	16	5	0	....
Bismarck	40	32	11	1.0	6.2
Milwaukee	45	25	7	0.5	....
Omaha	50	28	9	1.0	....
St. Louis	56	24	8	1.0	5.5
Chicago	48	25	7	0	....
Cincinnati	55	25	8	1.0	....
Denver	50	22	13	0	5.5
Pacific Coast States:					
Spokane	48	22	10	4.0	....
Portland, Oreg	53	16	8	3.0	3.2
San Francisco	56	5	6	0	....
Los Angeles	62	9	11	1.5	....
Arid West States:					
Boise	51	23	11	4.7	....
Santa Fe	49	22.5	11	1.5	3.5
Salt Lake City	51.6	22.2	10	2.5	4.0

[7] February 15 would be $\cos\left(180+\frac{\text{February 15–January 10}}{\text{July 30–January 10}}\times 180\right)=180+32°\ 24'=-\cos 32°\ 24'$, rather than $\cos\frac{360}{365}$ (February 15–July 30) $=\frac{360}{365}\times 200=\cos 197°=-\cos 17°$.

[8] The average daily variability of temperature for the year for the United States is 4.4° F. and for the arid west 4.7° F. (Bulletin Q, p. 33.) In the United States the maximum temperature occurs one-seventh of the time at an irregular time because of cyclones and the minimum one-fifth of the time at irregular times for the same reason. (Bulletin Q, p. 19.) *These facts give a slight indication of the departures of actual temperatures from the normal.*

[9] The equation gives the most accurate results in the spring and fall when the damaging frosts occur because the time of sunrise at these seasons is about the mean for the year, and this mean is used in the equation.